UPC 026

December 13, 1978
P. Limon

ELECTRON - PROTON INSTABILITIES IN THE ENERGY DOUBLER

I. Introduction

The circulating beam of the Energy Doubler will ionize residual gas molecules in the beam tube. The free electrons created in this process, if they are of sufficiently low energy, can be trapped in the beam potential well.[1] There are two possible instabilities that can be caused by these electrons, both of which are lumped under the general heading of neutralization.

1. A large number of electrons, traversed by the beam can cause a tune shift which might be large enough to cause a beam instability.[2]
2. Because of the initial momentum of the electrons, they will oscillate transversely in the potential well of the beam. The electromagnetic coupling of the electrons with the beam may result in unstable coherent oscillations of both beams.[3]
3. Tune shift

Following the arguments of Tigner[2], the partial derivitive of the space charge tune shift ΔQ_{sc} with respect to the neutralization fraction $\eta = \frac{Ne}{Np}$ is

$$\frac{\partial \, \Delta Q_{sc}}{\partial \, \eta} = \frac{N_p v_p R}{\pi b(a+b)Q(\frac{v}{c})^2}$$

where: N_p = the number of protons = 5×10^{13}

v_p = the classical proton radius = 1.5×10^{-18} m

R = the machine radius = 1000 m

a = half vertical beam size = $E_V \beta_V$

b = half horizontal beam size = $E_H \beta_H + \alpha^2 \left(\frac{\Delta p}{p}\right)^2$

Q = Tune value = 19.4

$v/c = 1$, $\gamma = E_p/m_p$

For the purposes of making estimates, we use the smooth approximation to the betatron functions,

$$\beta_H = \beta_V = R/Q = 51.5 \text{ m}$$

$$\alpha = R/\gamma_T^{\,2} = 3 \text{ m},$$

and

$$E_V = E_H = 13\pi/P(\text{GeV}) \text{ mm-mrad}$$

$$\Delta P/P = .02/p^{3/4} \text{ with the bunch spreader on.}$$

Then, for two energies, injection at 100 GeV and full energy at 1000 GeV

$$a = \begin{cases} 5.0 \text{ mm at 100 GeV} \\ 1.5 \text{ mm at 1000 GeV} \end{cases}$$

$$b = \begin{cases} 4.6 \text{ mm at 100 GeV} \\ 1.45 \text{ mm at 1000 GeV} \end{cases}$$

$$\frac{\partial \Delta Q}{\partial \eta} = \begin{cases} -.26 \text{ at 100 GeV} \\ -.27 \text{ at 1000 GeV} \end{cases}$$

So the rate of change of the tune shift with neutralization is almost energy independent.

If ΔQ max is the maximum allowable tune shift, then

$$\eta_{allowed} \approx \frac{\Delta Q max}{.27}$$

Conservatively, we can take $\Delta Q_{max} = 5 \times 10^{-3}$, then

$$\eta \lesssim 2 \times 10^{-2} \text{ is allowed.}$$

C. Coherent Electron-Proton Oscillations

Following the prescription of Schonauer and Zotter[3], we express the bounce frequency of the electrons and protons as multiples of the revolution frequency of the protons. The driving term of the oscillation is damped by spreads in the characteristic frequencies of both the protons and electrons. This spread is due to two factors:

1. The non-linear character of the potential well makes the bounce frequency amplitude dependent. We will ignore this term.
2. The changes in the size of the beam as a function of azimuth will change the bounce frequency of electrons. In the model used here, spread in the revolution frequency is ignored.

The stability criteria is

$$Q_p \lesssim \frac{8Q}{3\pi} \left(\frac{\Delta Q}{Q} \frac{\Delta Q_e}{Q_e} \right)^{1/2}$$

where

Q = proton betatron tune = 19.4

Q_e = electron bounce frequency

$$= \left(\frac{2 N_p r_e R}{\pi (v/c)^2 (a+b) b} \right)^{1/2}$$

Q_p = proton bounce frequency

$$= Q_e \left(\frac{M_e}{M_p} \frac{\eta}{\gamma} \right)^{1/2}$$

$\frac{\Delta Q}{Q}$ = betatron tune spread (chromaticity)

$\frac{\Delta Q_e}{Q_e}$ = electron bounce frequency spread (half)

taken as $\frac{Q_e^{max}-Q_e^{min}}{Q_e^{max}+Q_e^{min}}$, and where

we take a as a maximum at b_{min}, and vise-versa. For the β functions we use the tabulated amplitude functions in TM-797.[4] Since the momentum spread is a small effect on the beam size, we will use the smooth approximation of $\alpha = 3$ meters.

We then have the following parameters:

	β_H (m)	β_V (m)
F Quad	99.6	28.7
D Quad	28.7	99.6
Chromaticity	$\frac{\Delta Q}{Q}/\frac{\Delta P}{p}$ =	1.14

	100 GeV	1000 GeV
E_H mm-mrad	$.13\pi$	$.013\pi$
E_V mm-mrad	$.13\pi$	$.013\pi$
$\Delta P/P$	6.3×10^{-4}	1.13×10^{-4}
A_{max}mm	6.6	2.1
A_{min}mm	3.9	1.1
b_{max}mm	6.4	2.0
b_{min}mm	3.4	1.1
Q_eMax	1.6×10^3	3.14×10^3
$Q_e$min	1.2×10^3	3.75×10^3
$\Delta Q_e \pi Q_e$	.14	.16
Q_p Max	$3.6 \sqrt{\eta}$	$3.7 \sqrt{\eta}$
P_p min	$2.6 \sqrt{\eta}$	$2.7 \sqrt{\eta}$
$\Delta Q/Q$	7.2×10^{-4}	1.3×10^{-4}
Q_p threshold	1.65×10^{-1}	7.5×10^{-4}
η threshold	2.1×10^{-3}	7.7×10^{-4}

Clearly, the limit on is set by the coherent oscillation instability, and not by the tune shift. For our criterion, we take a limit of

$$\eta_{thresh} \lesssim 5 \times 10^{-4}.$$

D. Electron Formation

In the cold bore of the Energy Doubler, it is extremely difficult to determine the residual gas pressure. I will assume that it is all helium, at an equilibrium pressure of 10^{-10} torr, which corresponds to a surface coverage of about 1% of a monolayer around the whole ring. Certainly, the average pressure will be better than this, but some places might be worse.

For helium at STP

$$\frac{dE}{dx} = 1.94 \frac{\text{MeV}}{\text{g/cm}^2}, \quad \rho = 1.78 \times 10^{-4} \text{g/cm}^3,$$

and the energy loss/ion pair $\approx$30 EV. So

$$\frac{dN}{dx} \cong 12 \text{ ion pairs/cm at STP.}$$

At 4.3 K and 10^{-10} torr, the creation rate of electrons is

$$n_c = \frac{dN}{dx} \; 2\pi R \; f_o \cdot \frac{10^{-10}}{760} \; \frac{273}{4.3},$$

or n_c = 3 ion pairs/sec proton.

Some of these escape from the potential well, of course, but we will assume that all are captured. Threshold neutralization is reached in about

$$T = \frac{\eta_{thresh}}{N_c} = \frac{5 \times 10^{-4}}{3} = 160 \quad \text{secs},$$

or 8 revolutions of the beam.

E. Effect of Gaps in the Beam

When the beam has gaps, the force holding the electrons disappears periodically. For a gap of 15 ns, electrons of energy greater than 18 eV will hit the beam tube wall before the next bunch arrives. Some of these electrons will knock out other electrons from the wall, and can induce multipactoring. If the gap is long enough, however, all of the electrons will clear away, even those that are knocked off the wall with very low energy. A 100 ns gap, for example, will clear all electrons of 1 eV or higher energy. The injection system of the Energy Doubler will require a gap in the beam of some few hundred ns, which makes clearing automatic, at each turn. Since it takes roughly 8 turns to reach threshold, we are safe. One problem might occur if we stack beam in momentum space. In this case, it is difficult to keep a gap in the beam from filling up.

With the exception of this case, we believe we are safe from electron proton instabilities.

References

1. For effects due to the positively charged ions repelled by the beam see: "The Pressure Bump Instability in the Fermilab Energy Doubler", P. Limon, UPC No. 3, Fermilab November 1978.

2. "Estimates Concerning Beam Neutralization for POPAE" M. Tigner, TM-521, Fermilab, 1974.

3. "Electron-Proton Instabilities in the SPS", H. Schonauer and B. Zotter, Spring Study on Accelerator Theory, AMC-1, June, 1972.

4. "Doubler Lattice and Phase-Amplitude Functions", T.L. Collins, TM-797, Fermilab, June, 1978.